
Statistical Process Control Concepts

Flavio Giurco

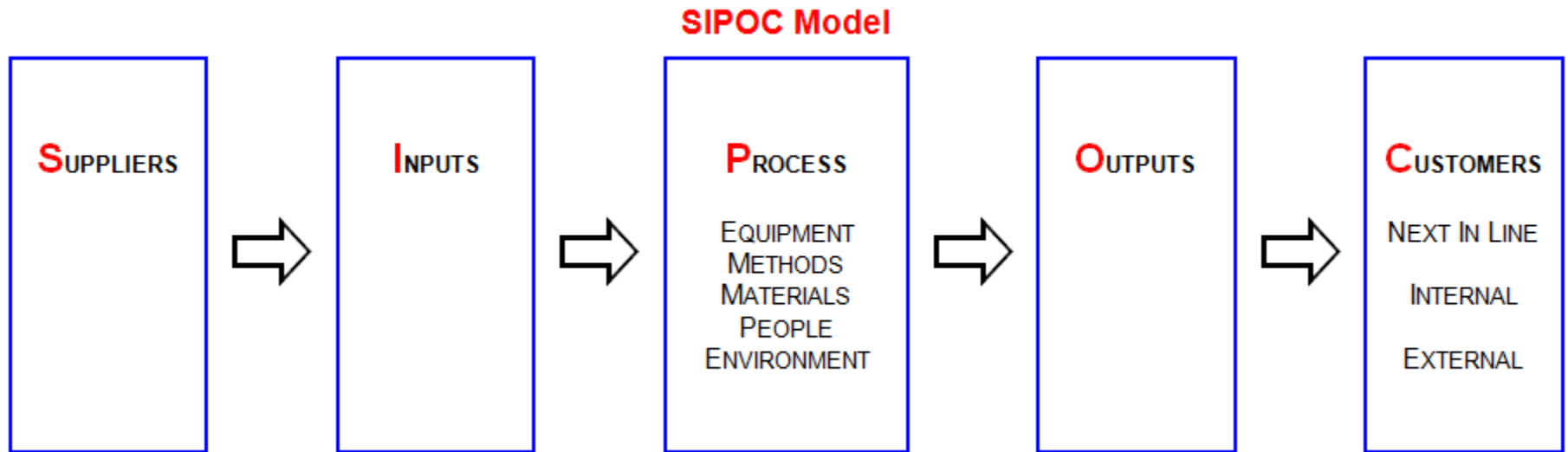
**Geelong Quality Council
1-Oct-2015**

Introduction

The focus of this presentation is on:

- Concepts and Understanding
- What is Statistical Process Control?
- The meaning of various basic statistics
- Elements and interpretation of control charts
- Understand common causes and special causes of variation
- How understanding different types of variation helps direct our problem solving
- Understand the difference between control limits and specification limits
- Become aware of the various types of control charts
- Displaying variation using Box & Whisker plots
- Ideas on “Putting it all together” for comprehensive monitoring of performance

Statistical Process Control



Basic Statistics To Describe Population

POPULATIONS

Samples are taken to estimate what the population looks like.

Populations can be described by:

LOCATION / CENTRAL TENDENCY

- Mean = average value
- Median = middle value

SPREAD / DISPERSION

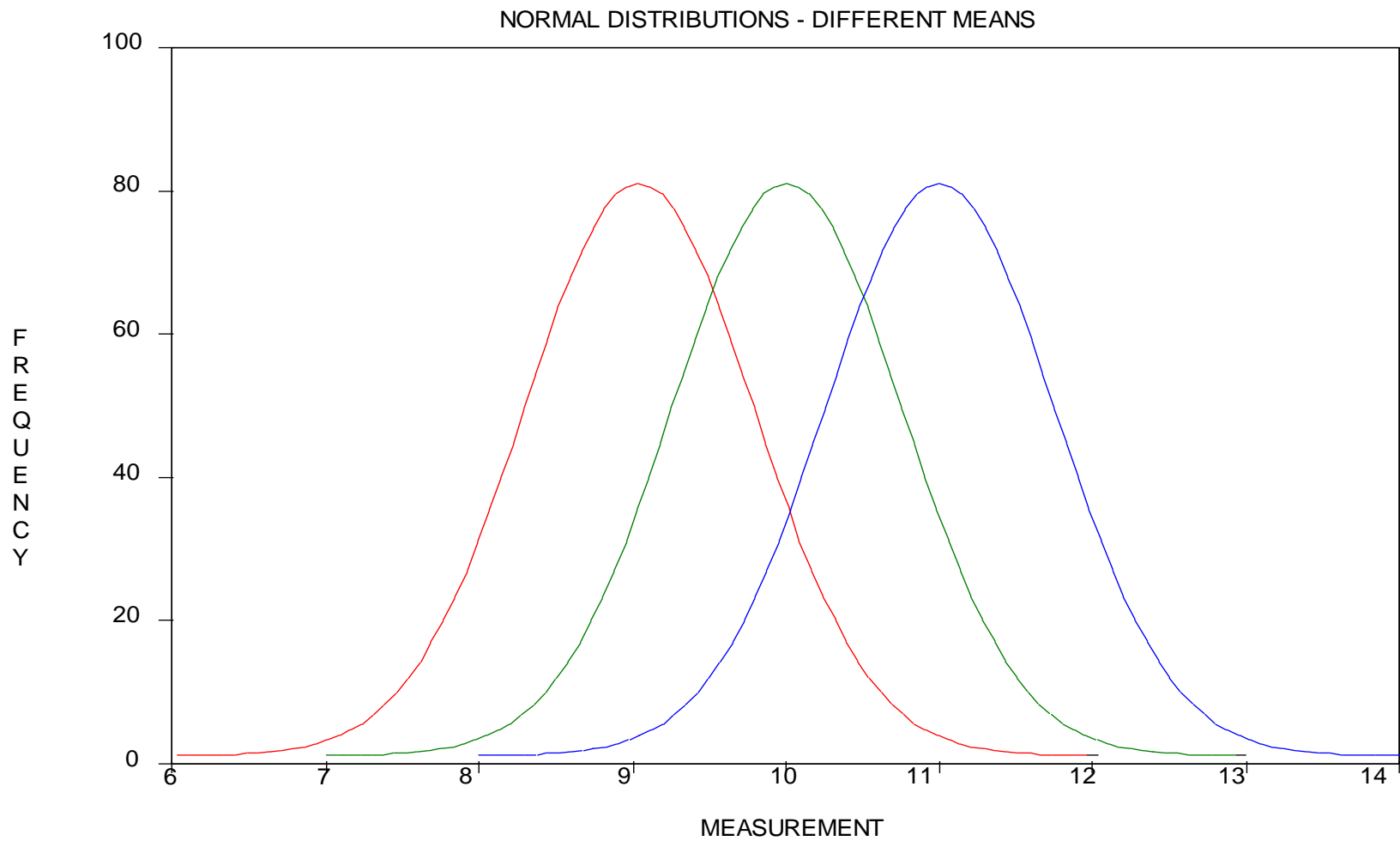
- Range = maximum – minimum
- Standard Deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

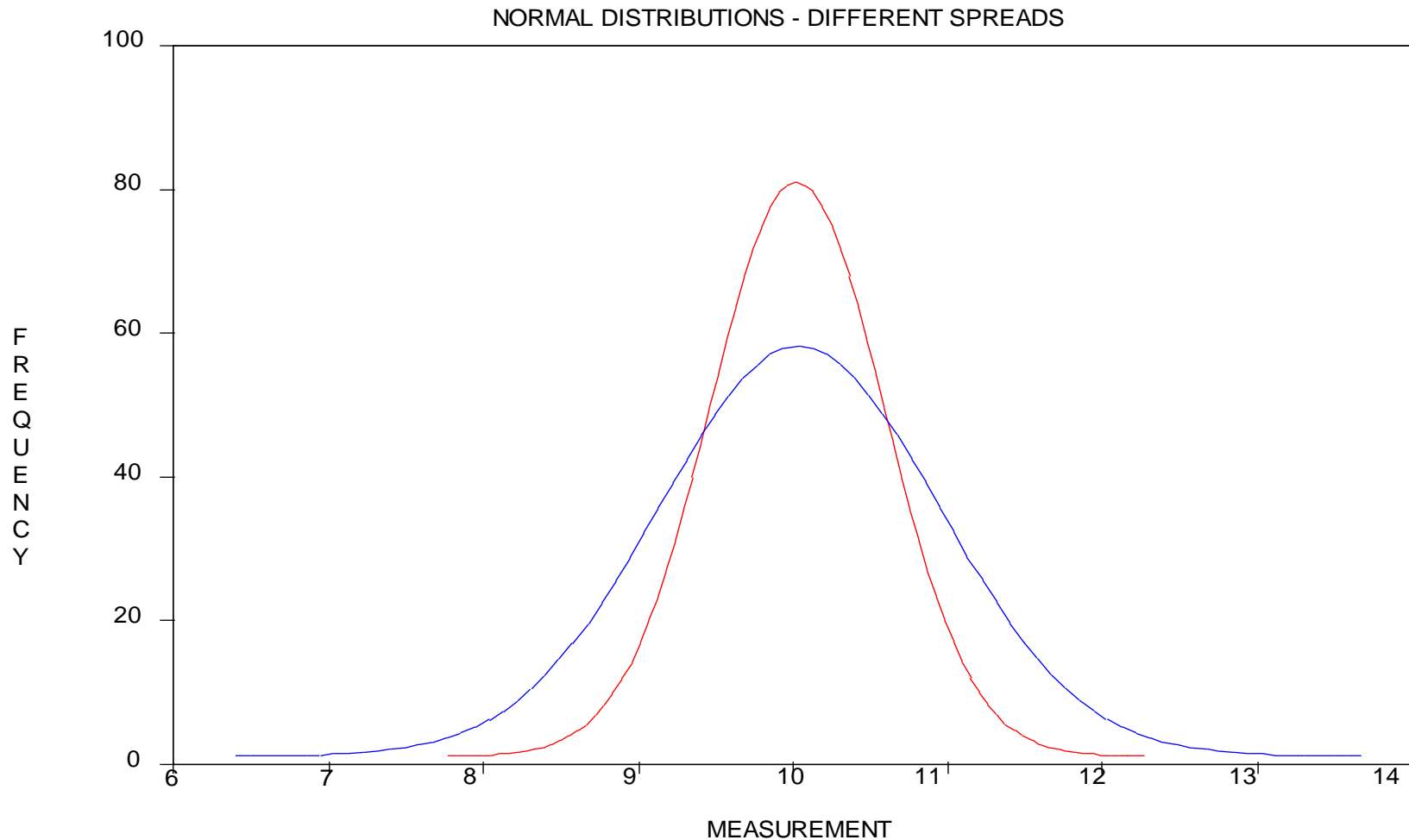
SHAPE

- Skewness = how symmetric
- Kurtosis = how “pointed”

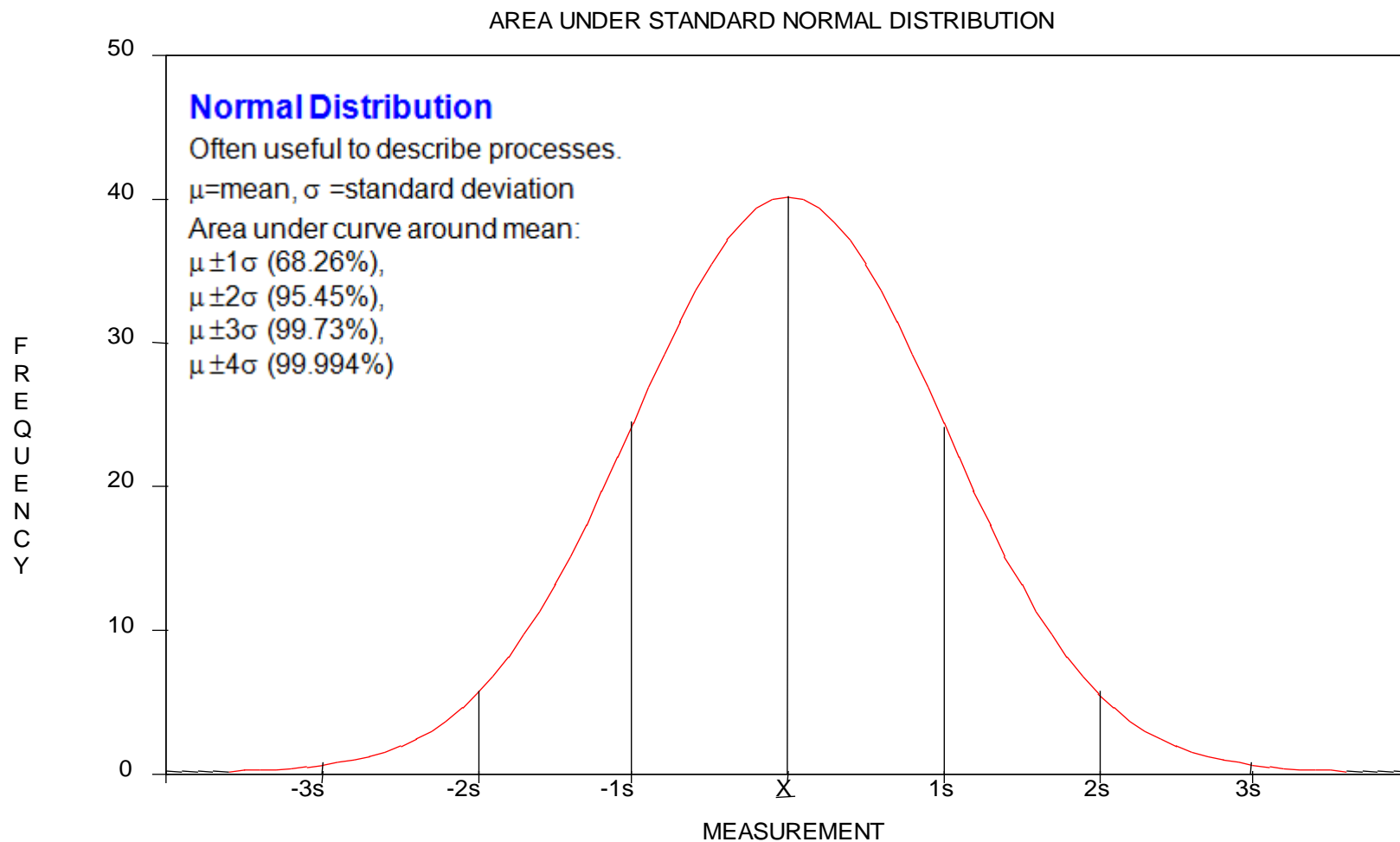
Location - Different Means



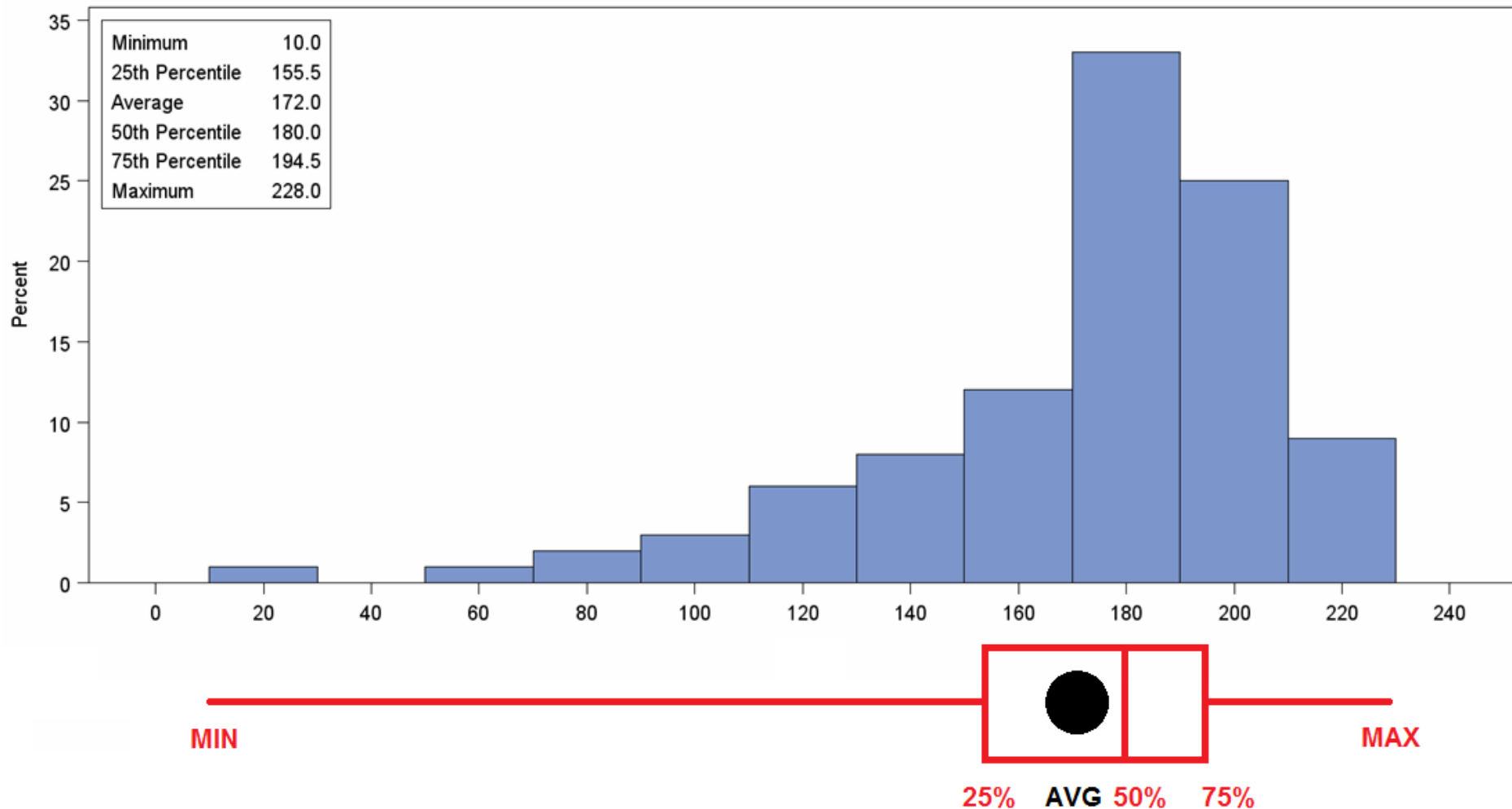
Different Spreads (Standard Deviation)



Normal Distribution



Histogram / Percentiles & Box and Whiskers



Understanding Variation

UNDERSTANDING VARIATION

Variability exists, we expect items to vary!

We need to understand variability.

Statistics helps describe and understand variability.

Types of Causes of Variation

COMMON CAUSES

= usual variability, nothing unusual, part of the "system"

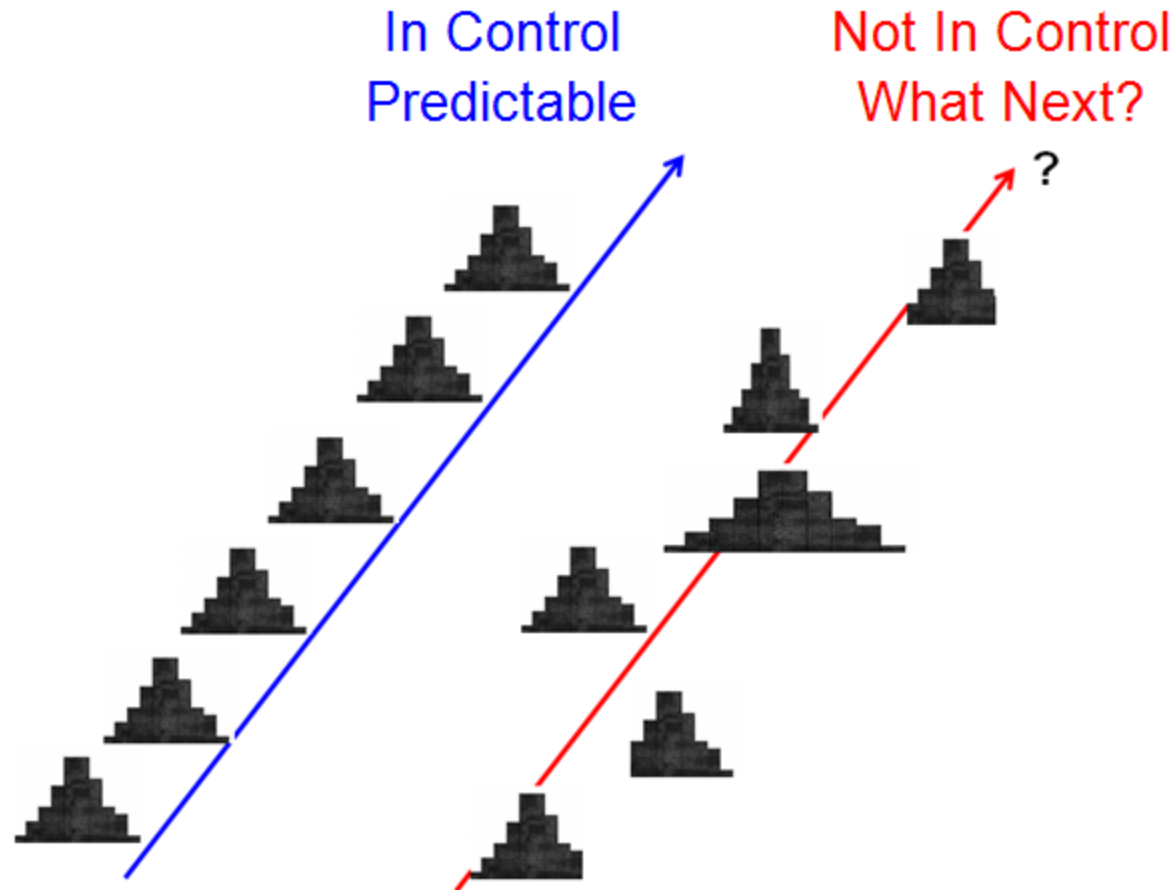
= lots of things each having a small impact

SPECIAL CAUSES

= something out of the ordinary, unusual

= smaller number of things having a large impact

"Control" = predictable without excursions



Elements of a Control Chart

Control Charts

Elements

CENTRE LINE

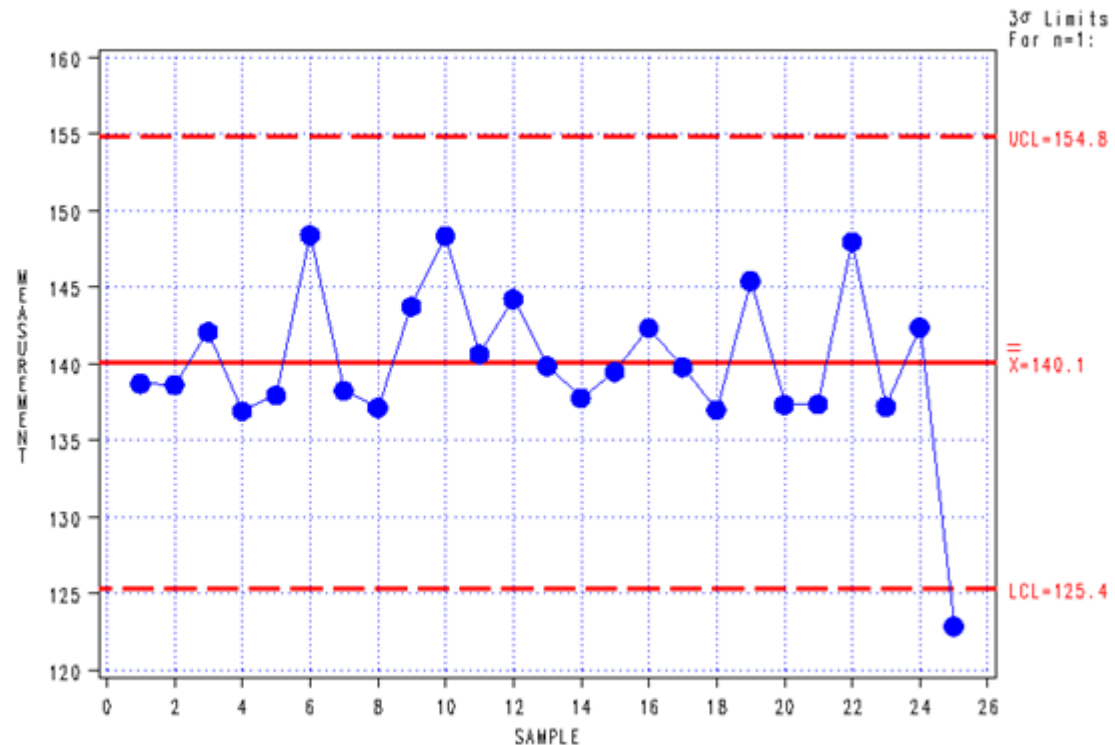
eg $\bar{\bar{X}}$, \bar{R} , \bar{S}

UCL

= Upper Control Limit

LCL

= Lower Control Limit



Control limits, in the context of Control Charts, are calculated from the data reflecting expected variability. They represent “Bounds of Belief”.

Specification limits are set by the Customer and represent what are the desired outcomes from your process.

Types of Control Charts

CONTINUOUS DATA

eg temperature, weight, tensile strength

X-bar and R chart (mean and range)

X-bar and S chart (mean and standard deviation)

X and Rm chart (individual and moving range)

M and R chart (median and range)

ATTRIBUTE DATA

eg broken/not broken, good/bad

P chart (proportion)

nP chart (proportion number)

C chart (count)

U chart (count per unit)

SPECIAL CHARTS

EWMA chart (exponentially weighted moving average)

CUSUM chart (cumulative sum)

BOX chart (Box and Whisker plots)

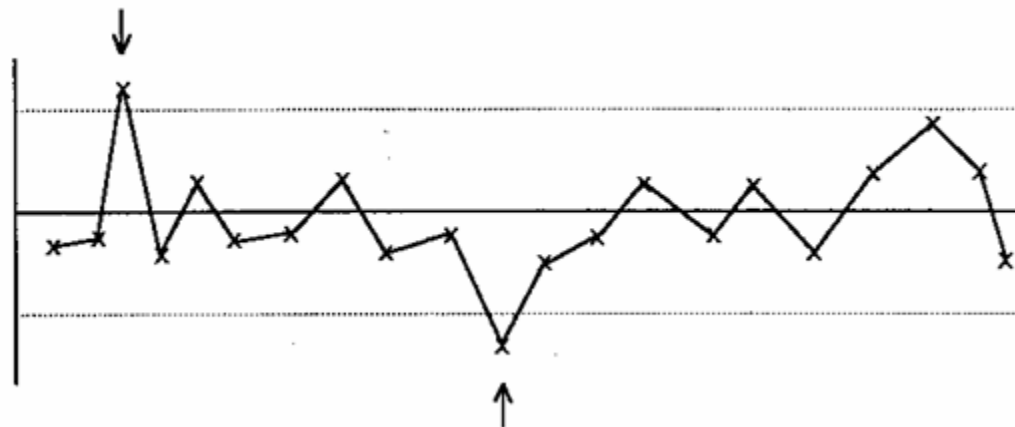
Guidelines For Assessing "Control".

A process is considered to be in a state of statistical control if none of the following rules are violated.

The following 7 "Rules" describe conditions which indicate a possible change in the process.

The rules provide an agreed, objective method to decide if the process may have changed and may need to be adjusted or if the process has not changed and should be left alone.

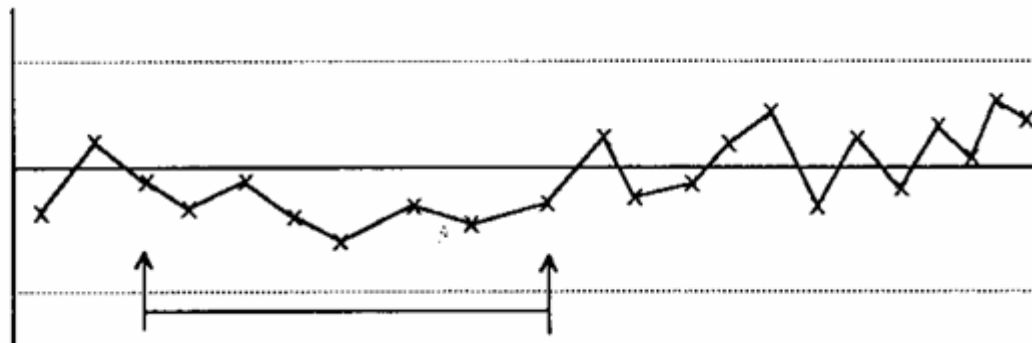
Rule 1
**POINT BEYOND
CONTROL LIMITS**



Exceptions Rules 2 - 4

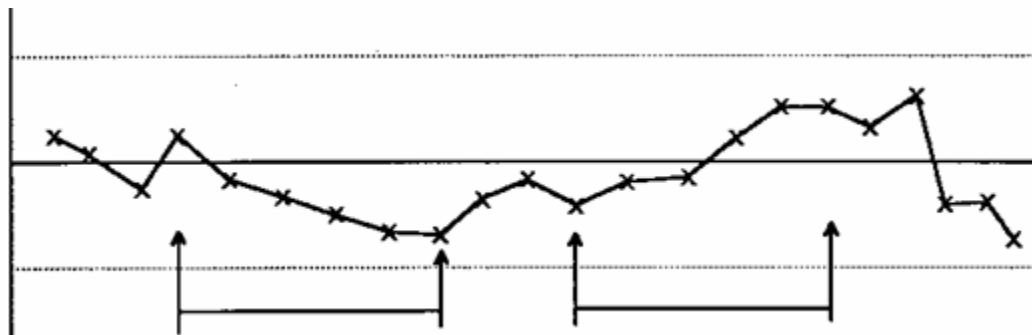
Rule 2

RUN - 8 POINTS IN A
ROW ON ONE SIDE OF
CENTRE LINE



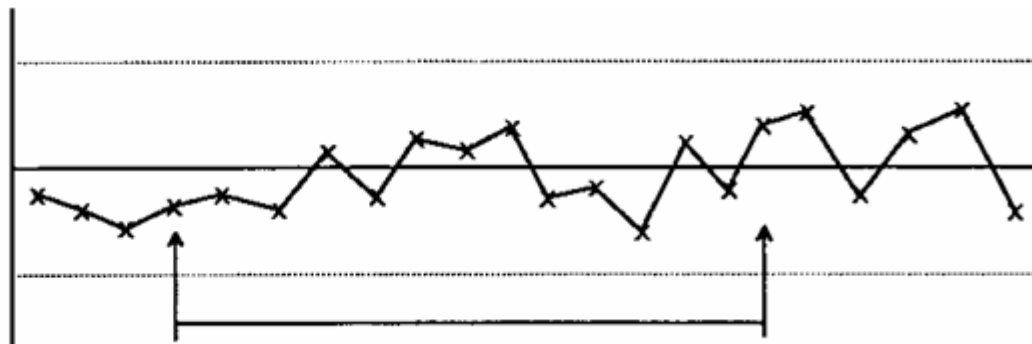
Rule 3

TREND - 6 POINTS IN A
ROW INCREASING OR
DECREASING



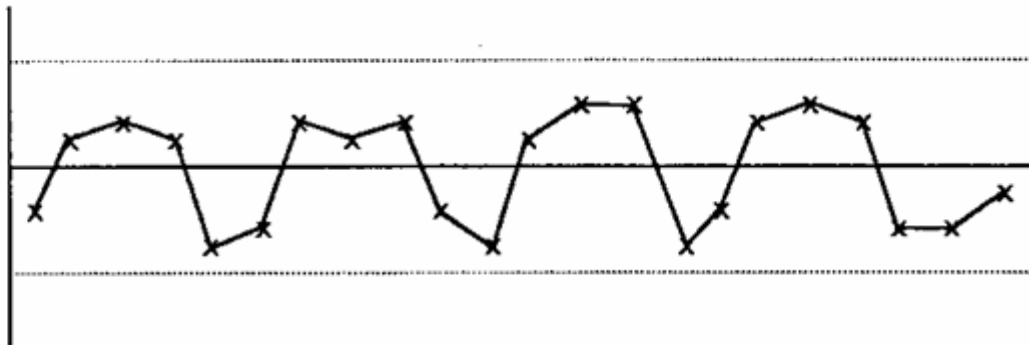
Rule 4

FOURTEEN POINTS IN A
ROW ALTERNATING UP
& DOWN

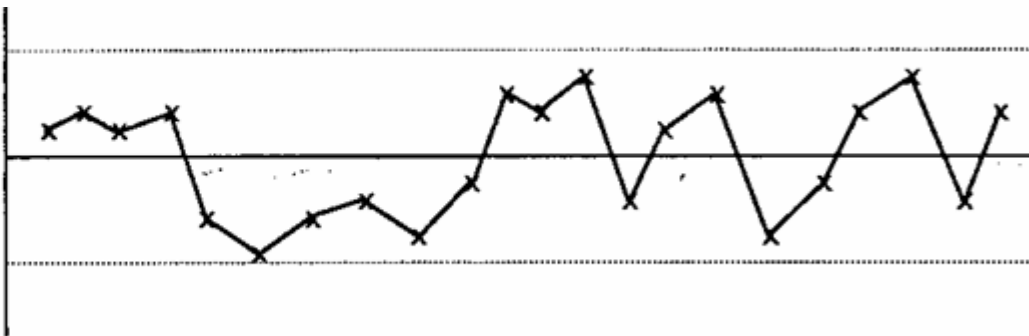


Exceptions Rules 5 - 7

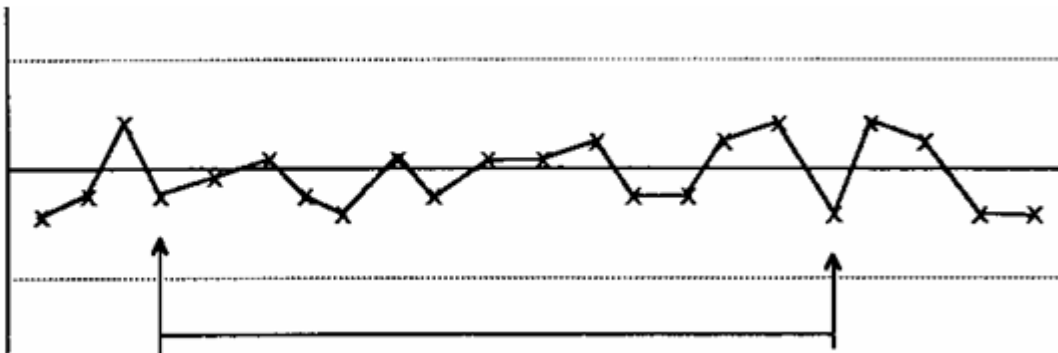
Rule 5
CYCLES



Rule 6
**EXCESSIVE
VARIABILITY**

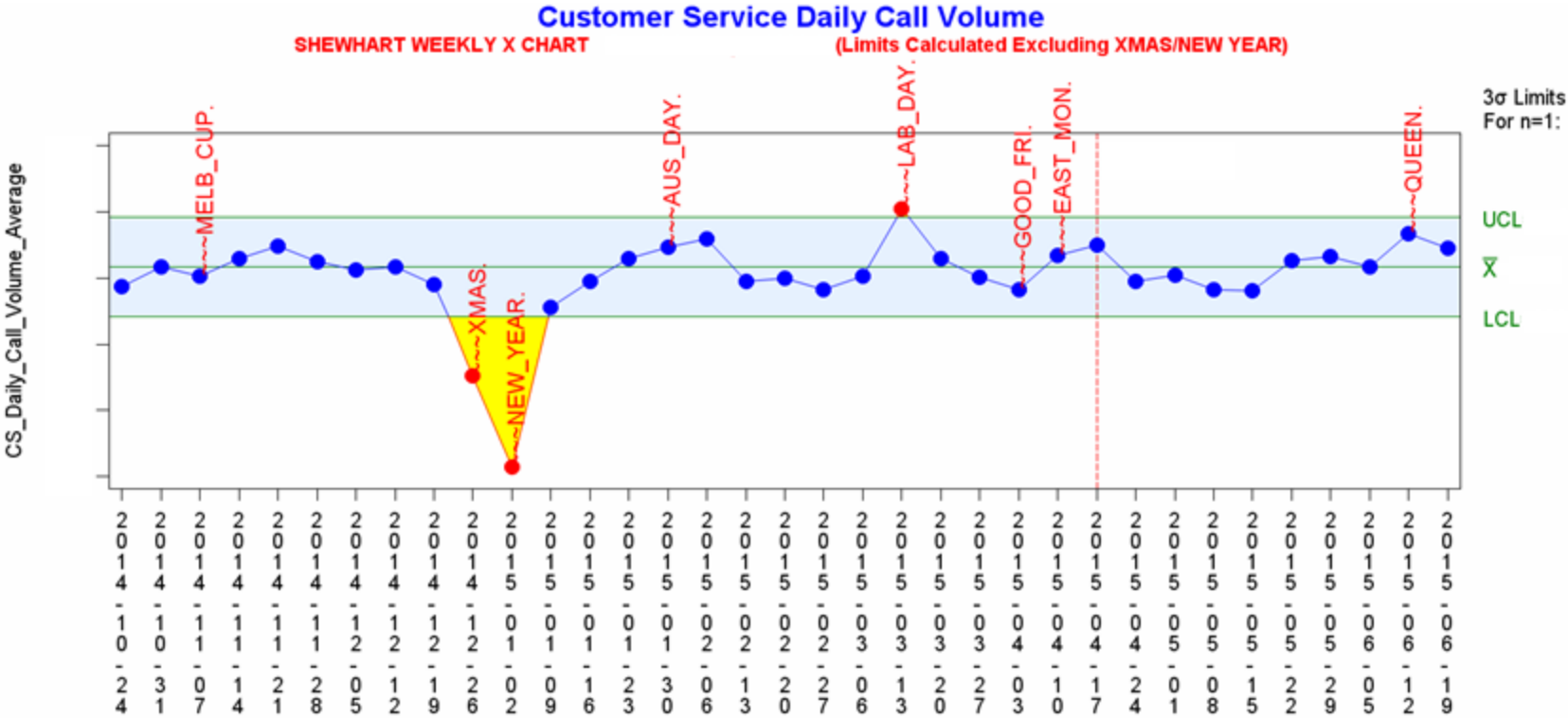


Rule 7
LACK OF VARIABILITY



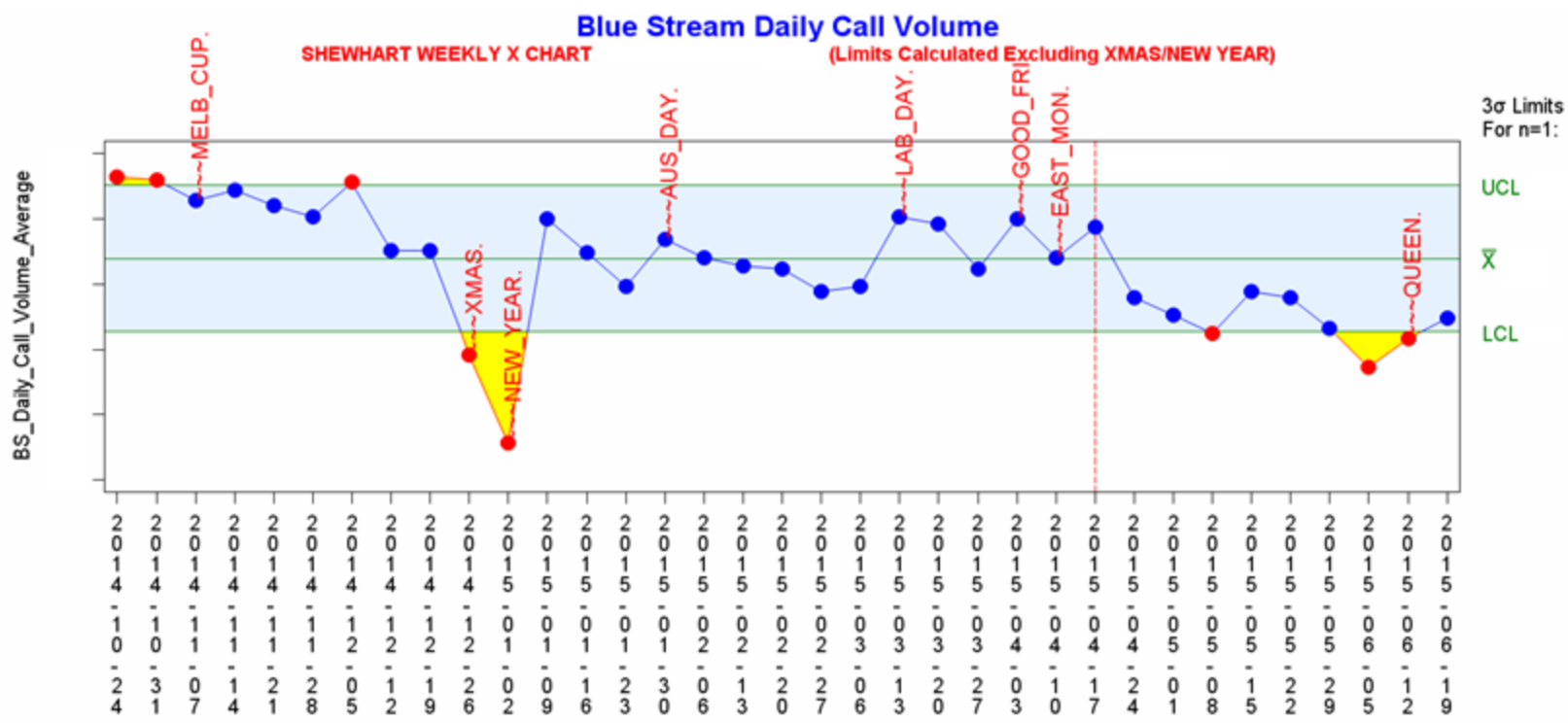
Case #1

Daily Calls (Customer Service) by Week



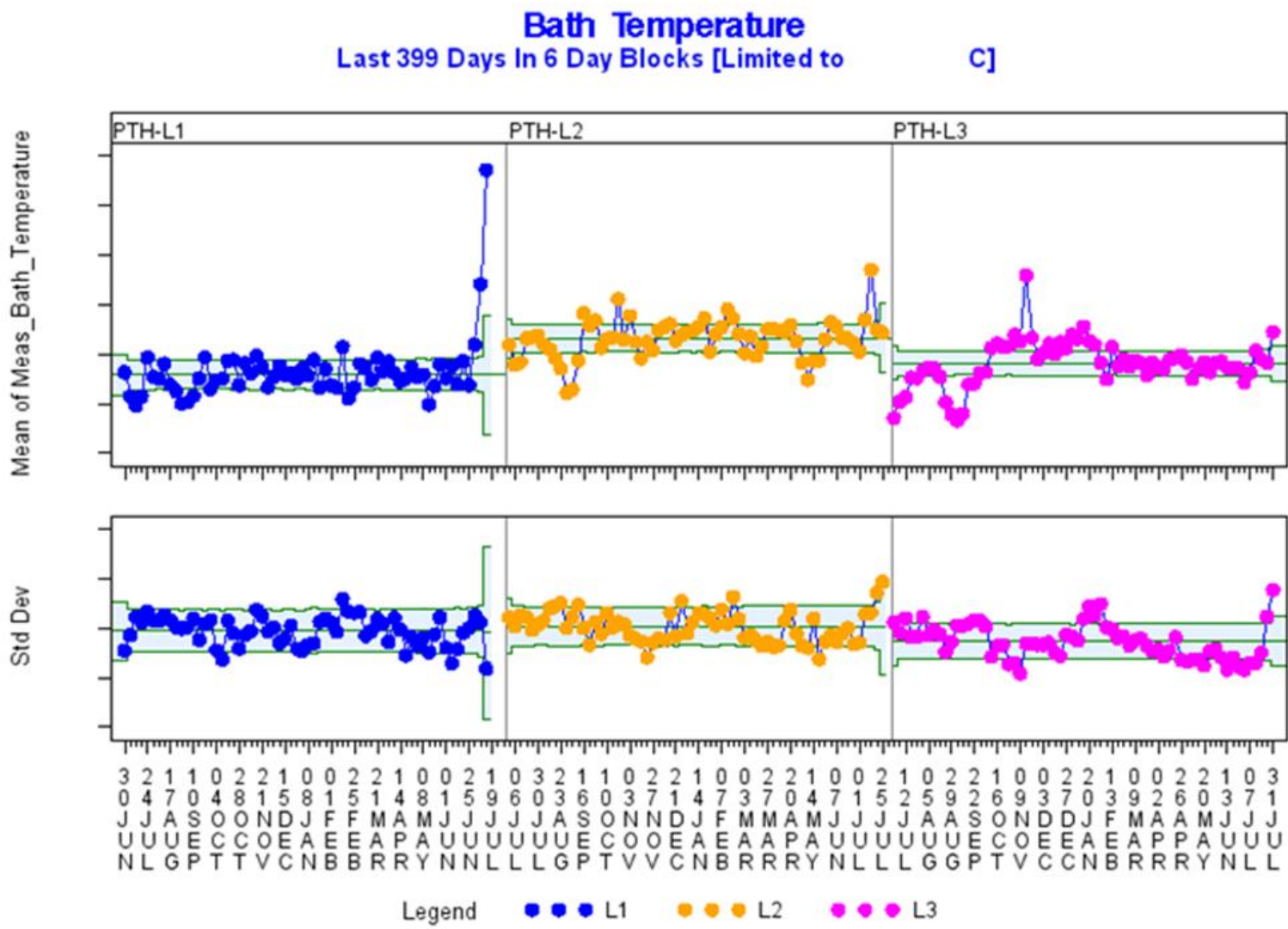
Case #2

Daily Calls (Blue Stream) by Week



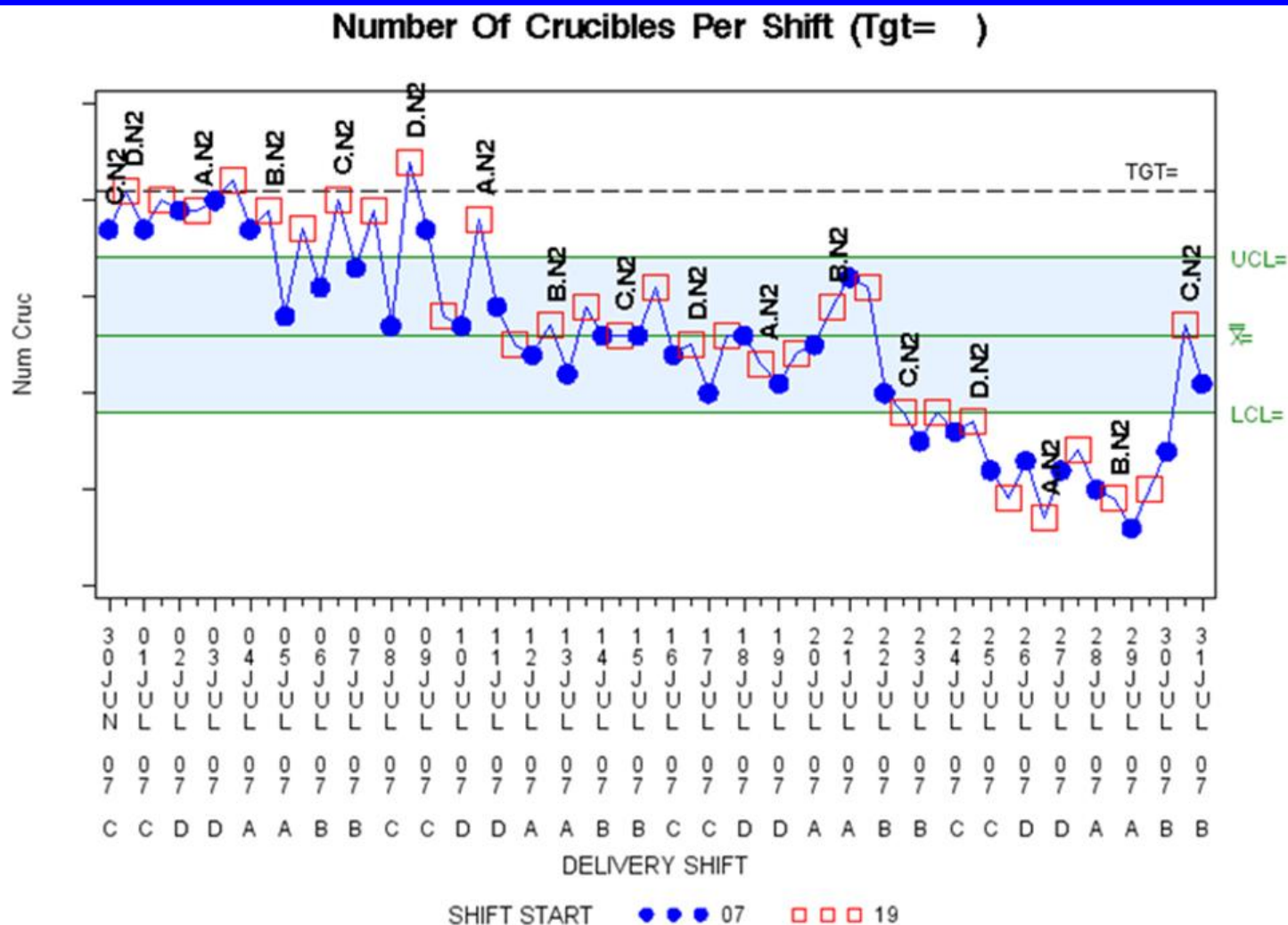
Case #3

X-bar & S chart (3 lines) by 6 Days



Case #4

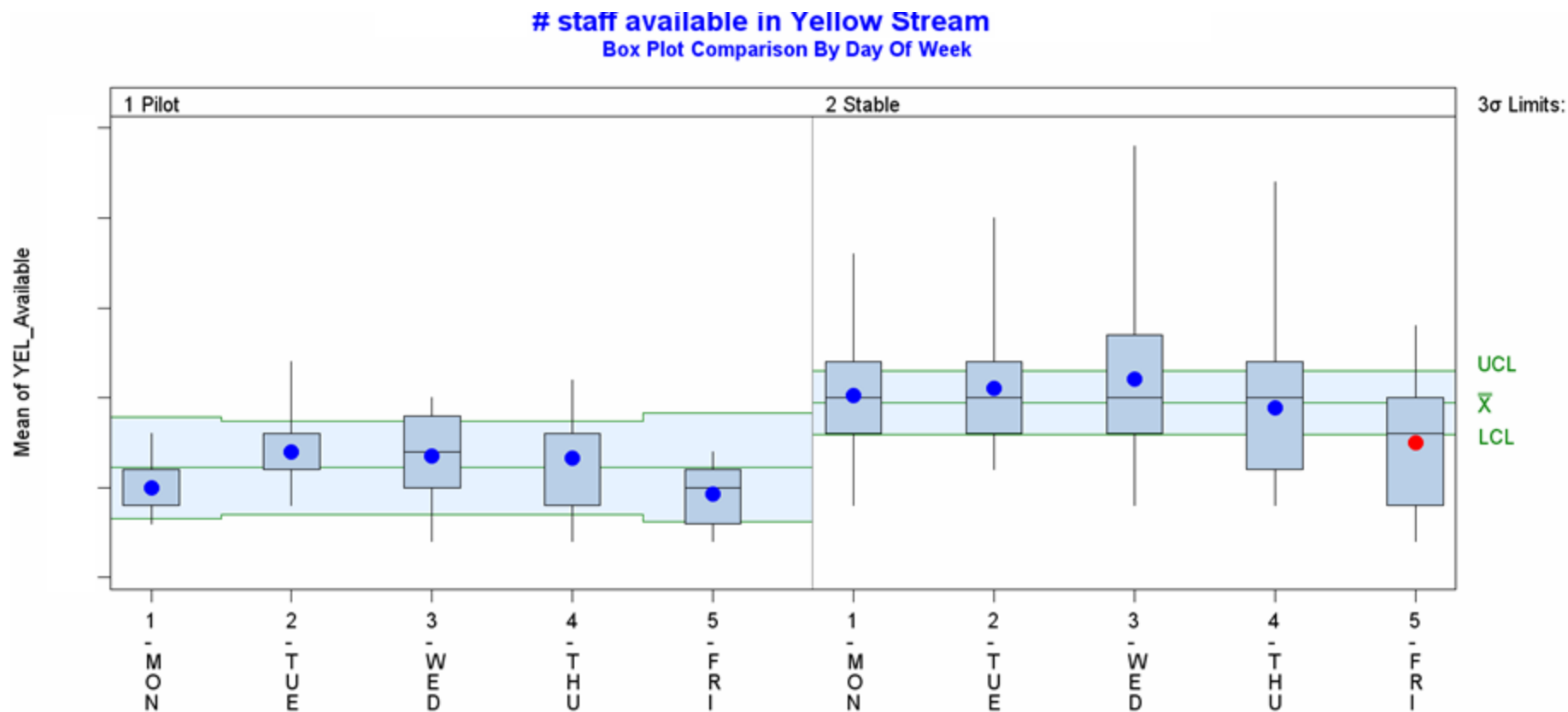
Metal Flow - Crucibles by Shift



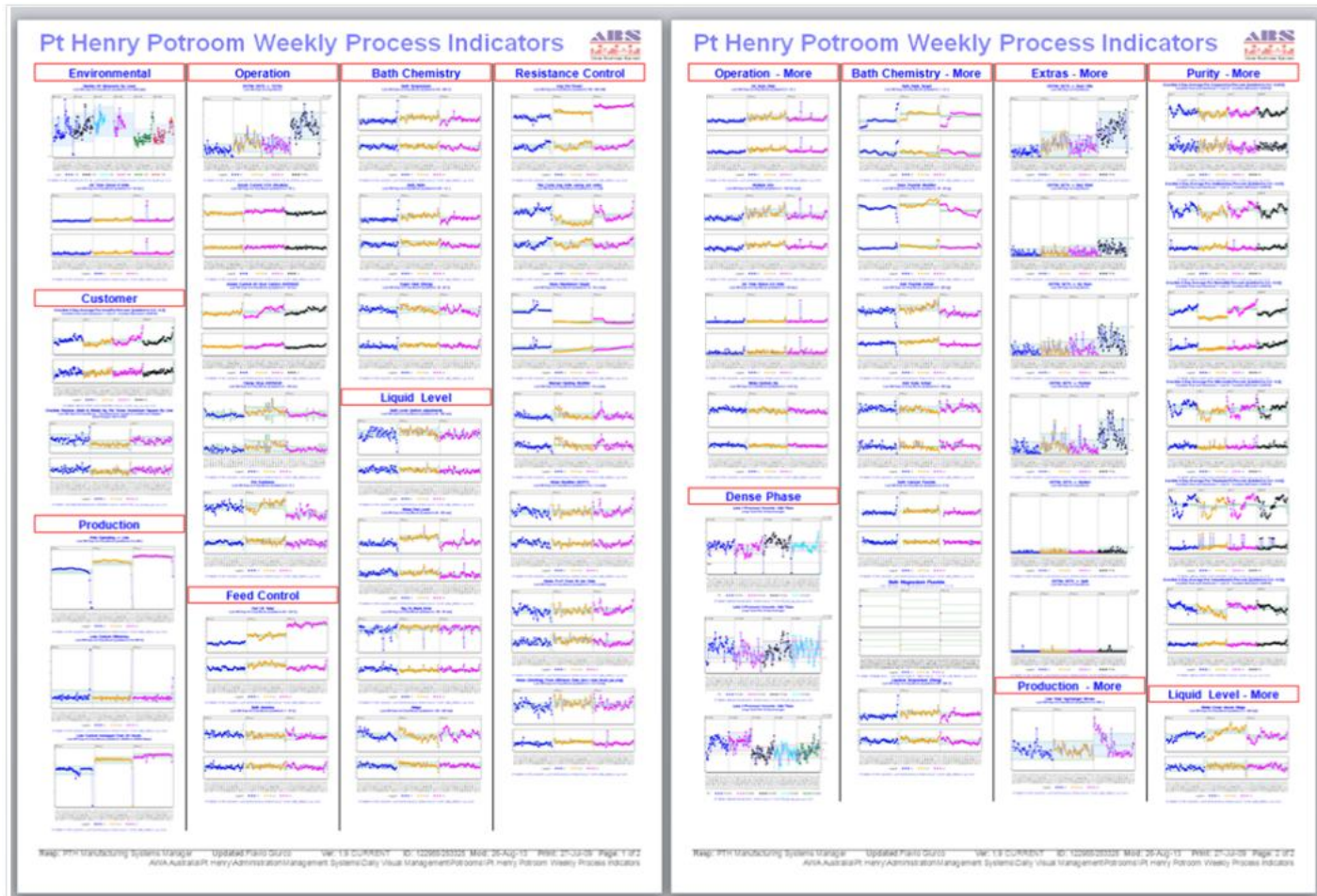
- Annotated by CREW and Second Night Shift

Case #5

Box & Whisker Charts by Day of Week



Technical Overview - Word A3 Mini Plots



Factors affecting pot temperature:

- Pot Voltage
- Line Current
- Bath Composition
- Metal Level
- Bath Level
- Pot "Noise"
- Anode Failures
- Anode Loading

Small plots of a range of variables make it easier to see any common trends.

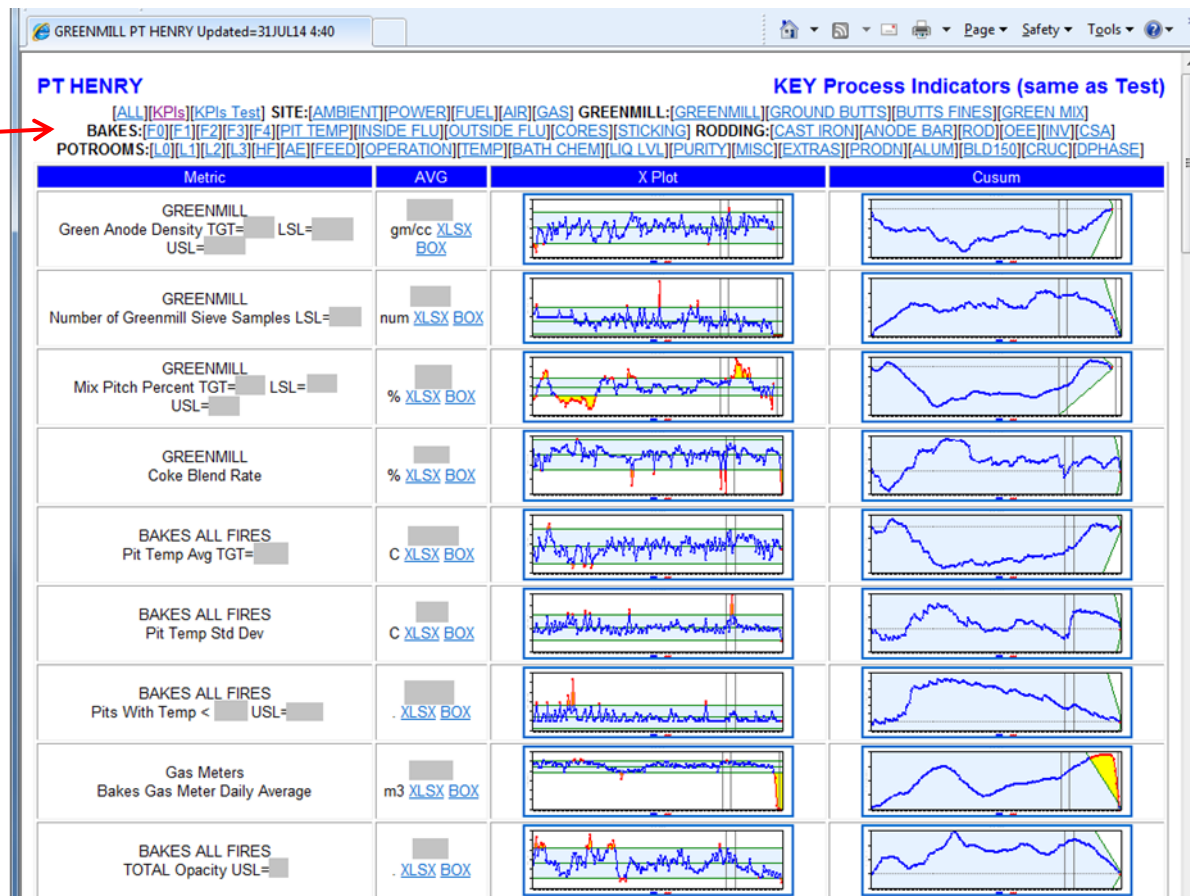
- Double sided A3 paper with 28 metrics per page.
- Printed and reviewed daily by potroom process engineer.

Management Review

- Managers' Monday morning meeting.
- Managers and technical resources from across smelting departments.
- Catalyst for development: Major Process excursion
- Look outside one's own department!
- 53 KPIs from 566 Metrics:
 - Greenmill (4),
 - Bakes (10)
 - Anode Bar Repair (1)
 - Rodding (2)
 - Electrode (4)
 - Customer Supplier Survey (2)
 - Potrooms (27)
 - Compressed Air (1)
 - Power (1)
 - Fuel (1)
- "Time Shift" of data to when material goes to potrooms

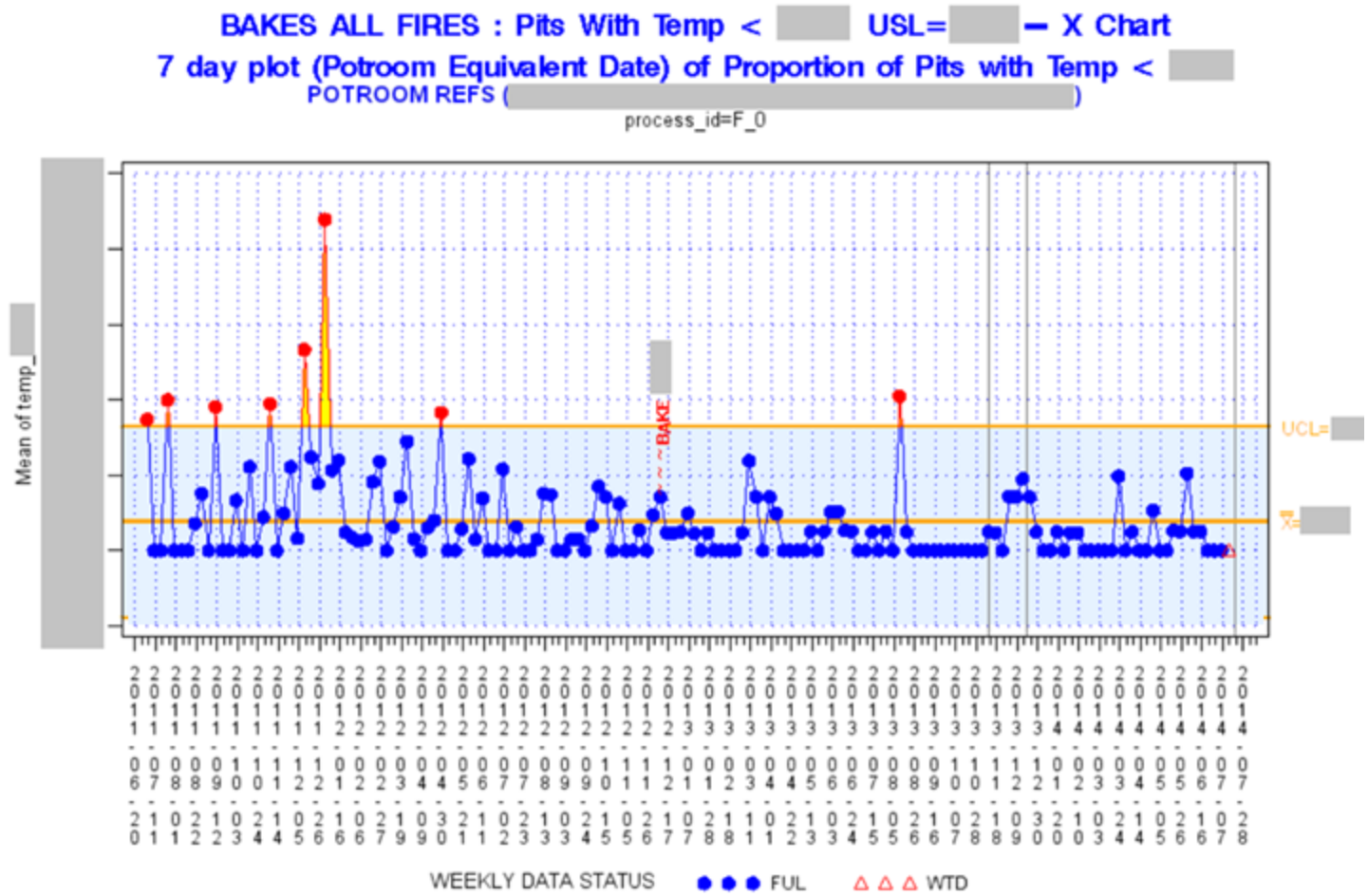
Key Performance Indicators on Intranet

Links to
other pages



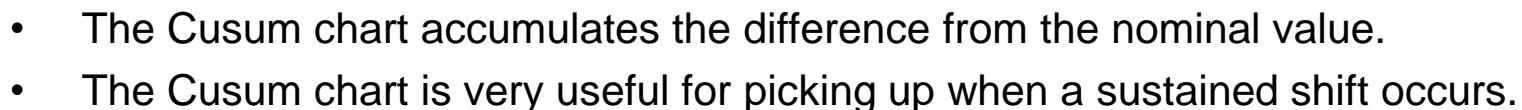
- Miniature plots for quick scanning. X chart and Cusum chart (Click to enlarge)
- Menu via links to web pages for each class of metric.
- Links to Excel data (XLSX) and Box plots

Long Term Charts Over 3 Years by Week



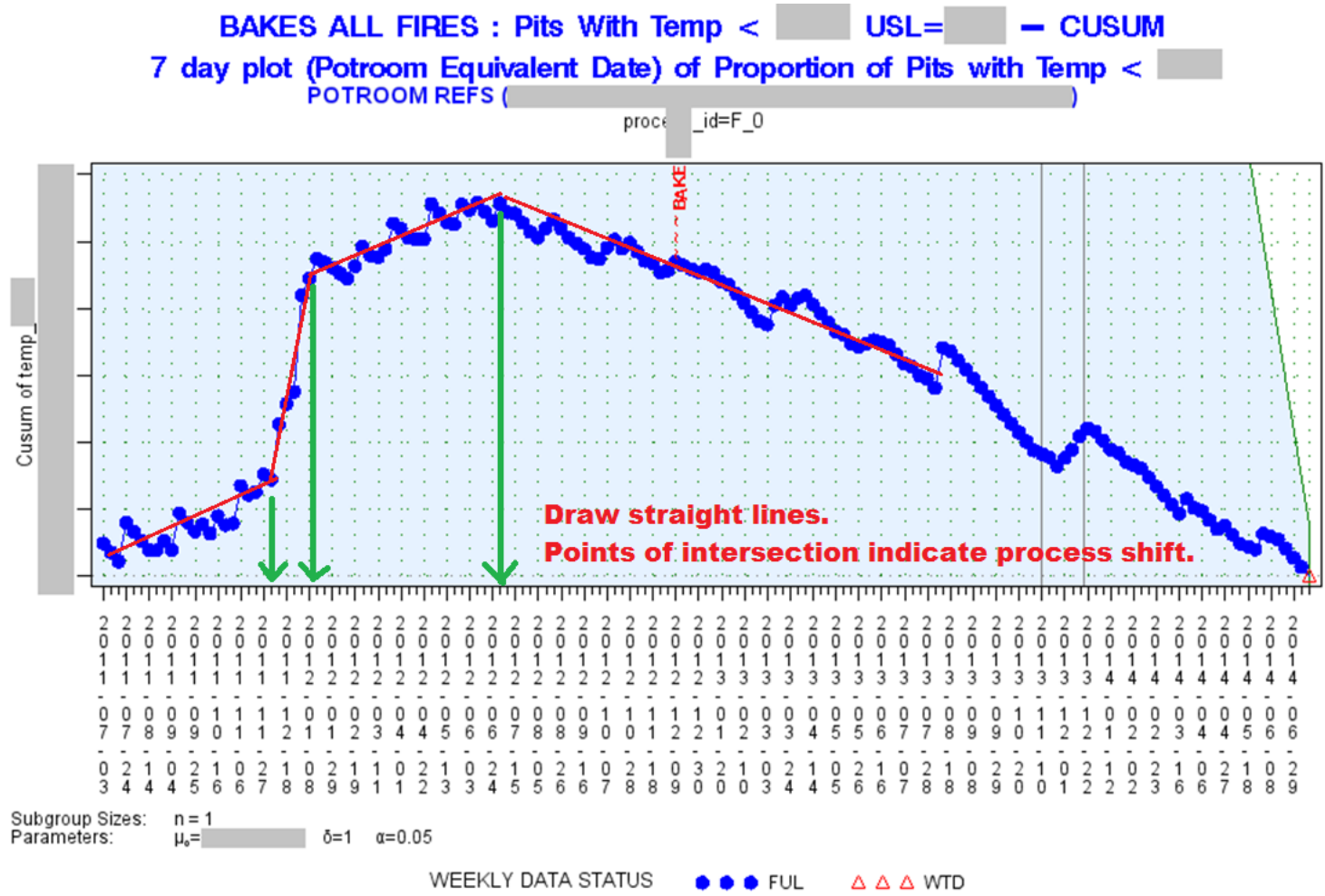
Going Forward

- Today's focus was **understanding** the Concepts of Statistical Process Control.
- The Appendix in the following pages provides more details.
- Use control charts to monitor your process metrics.
- Make the charts readily available to all personnel.
- Base your decisions on data with an understanding of variability.
- Build in a formal periodic review of performance metrics.
- **Start small and grow/evolve where there is demand / biggest pay back!**



Appendix

- Cusum Chart Part 2 of 2



Appendix

- Components of Variation

Manufactured products have 3 components of variation.

When designing a sampling and measurement plan, these components need to be considered.

WITHIN PIECE:

A characteristic measured at multiple points on the same part may be different at each point. For example, the coating weight across the width of a sheet of metal may vary slightly.

PIECE TO PIECE WITHIN A LOT:

A characteristic measured on many parts from the same lot may be different on each part. For example, the gauge from several coils from the same lot may vary.

LOT TO LOT:

Typically, a characteristic of the product will vary from shift to shift, day to day, setup to setup, etc.

Appendix

- Data Collection Considerations

- Data collected for establishing the control limits must be gathered and recorded according to a prescribed plan.
- The process must be sampled using a predetermined frequency and sample size.
- An absolute minimum of 15 sets of data are needed to establish control limits, preferably 25 sets.
- The sets of data must span enough production time so that the natural variation associated with the process or machine will be observed.

Acknowledgements:

Some of the material used in this presentation has been sourced from materials prepared by Michael Mazu and by Greg Peterson.

Appendix

- Creating Control Charts

- **COMMON STEPS TO CREATE ANY CONTROL CHART:**
- Collect data.
- Calculate appropriate statistics.
- Plot the appropriate statistics in time sequence.
- Calculate appropriate control limits.
- Draw control limits on the control chart.
- Check for control.
- If process is in control, establish an ongoing plan.
- If process is out of control then
 - (a) find causes,
 - (b) adjust process and
 - (c) collect more data.

Appendix

- Charts for Variables & Attributes

CONTROL CHARTS FOR VARIABLES

Used in pairs: one chart for controlling the average performance and one chart for controlling the variability of the process. Typical pairs:

- X-Bar and Range control charts
- X-Bar and Standard Deviation control charts
- Individuals and Moving Range control charts

Variables Chart	Centre line	Lower Limit	Upper Limit
X-Bar (with R)	$\bar{\bar{X}}$	$\bar{\bar{X}} - A_2 \bar{R}$	$\bar{\bar{X}} + A_2 \bar{R}$
X-Bar (with S)	$\bar{\bar{X}}$	$\bar{\bar{X}} - A_3 \bar{s}$	$\bar{\bar{X}} + A_3 \bar{s}$
Median (with R)	$\bar{\bar{M}}$	$\bar{\bar{M}} - 1.15 A_2 \bar{R}$	$\bar{\bar{M}} + 1.15 A_2 \bar{R}$
Individual	$\bar{\bar{X}}$	$\bar{\bar{X}} - E_2 \bar{MR}$	$\bar{\bar{X}} + E_2 \bar{MR}$
Range	\bar{R}	$D_3 \bar{R}$	$D_4 \bar{R}$
Stand. Dev.	\bar{s}	$B_3 \bar{s}$	$B_4 \bar{s}$

where:

- $\bar{\bar{X}}$ = the average of the results
- $\bar{\bar{X}}$ = the average of the averages (the grand average)
- $\bar{\bar{M}}$ = the grand median
- \bar{R} = the average range
- \bar{s} = the average standard deviation
- \bar{MR} = the average moving range

CONTROL CHARTS FOR ATTRIBUTES

Attributes Chart	Centre line	Lower Limit	Upper Limit
% Defective	\bar{p}	$\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$	$\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$
# Defectives	$n\bar{p}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
# Defects	\bar{c}	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{c} + 3\sqrt{\bar{c}}$
Defects/unit	\bar{u}	$\bar{u} - 3\sqrt{\bar{u}/n}$	$\bar{u} + 3\sqrt{\bar{u}/n}$

where:

- \bar{p} = the average proportion defective
- $n\bar{p}$ = the average number of defectives
- \bar{c} = the average number of defects
- \bar{u} = the average number of defects per unit

Appendix

- Control Chart Formulae & Factors

CONTROL CHART FORMULAE

Variables Chart	Centre line	Lower Limit	Upper Limit
X-Bar (with R)	$\bar{\bar{X}}$	$\bar{\bar{X}} - A_2 \bar{R}$	$\bar{\bar{X}} + A_2 \bar{R}$
X-Bar (with S)	$\bar{\bar{X}}$	$\bar{\bar{X}} - A_3 \bar{s}$	$\bar{\bar{X}} + A_3 \bar{s}$
Median (with R)	$\bar{\bar{M}}$	$\bar{\bar{M}} - 1.15 A_2 \bar{R}$	$\bar{\bar{M}} + 1.15 A_2 \bar{R}$
Individual	$\bar{\bar{X}}$	$\bar{\bar{X}} - E_2 \bar{MR}$	$\bar{\bar{X}} + E_2 \bar{MR}$
Range	\bar{R}	$D_3 \bar{R}$	$D_4 \bar{R}$
Stand. Dev.	\bar{s}	$B_3 \bar{s}$	$B_4 \bar{s}$
Attributes Chart	Centre line	Lower Limit	Upper Limit
% Defective	\bar{p}	$\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$	$\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$
# Defectives	$n\bar{p}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
# Defects	\bar{c}	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{c} + 3\sqrt{\bar{c}}$
Defects/unit	\bar{u}	$\bar{u} - 3\sqrt{\bar{u}/n}$	$\bar{u} + 3\sqrt{\bar{u}/n}$

TABLE OF CONTROL CHART FACTORS

n	A ₂	A ₃	E ₂	B ₃	B ₄	D ₃	D ₄
2	1.880	2.659	2.660	0.000	3.267	0.000	3.267
3	1.023	1.954	1.772	0.000	2.568	0.000	2.574
4	0.729	1.628	1.457	0.000	2.266	0.000	2.282
5	0.577	1.427	1.290	0.000	2.089	0.000	2.114
6	0.483	1.287	1.184	0.030	1.970	0.000	2.004
7	0.419	1.182	1.109	0.118	1.882	0.076	1.924
8	0.373	1.099	1.054	0.185	1.815	0.136	1.864
9	0.337	1.032	1.010	0.239	1.761	0.184	1.816
10	0.308	0.975	0.975	0.284	1.716	0.223	1.777
11	0.285	0.927		0.321	1.679	0.256	1.744
12	0.266	0.886		0.354	1.646	0.283	1.717
13	0.249	0.850		0.382	1.618	0.307	1.693
14	0.235	0.817		0.406	1.594	0.328	1.672
15	0.223	0.789		0.428	1.572	0.347	1.653
16	0.212	0.763		0.448	1.552	0.363	1.637
17	0.203	0.739		0.466	1.534	0.378	1.622
18	0.194	0.718		0.482	1.518	0.391	1.608
19	0.187	0.698		0.497	1.503	0.403	1.597
20	0.180	0.680		0.510	1.490	0.415	1.585
21	0.173	0.663		0.523	1.477	0.425	1.575
22	0.167	0.647		0.534	1.466	0.434	1.566
23	0.162	0.633		0.545	1.455	0.443	1.557
24	0.157	0.619		0.555	1.445	0.451	1.548
25	0.135	0.606		0.565	1.435	0.459	1.541